9.2: 18, 26

18. $\mathbf{a}+\mathbf{b}=(2\,\mathbf{i}-4\,\mathbf{j}+4\,\mathbf{k})+(2\,\mathbf{j}-\mathbf{k})=2\,\mathbf{i}-2\,\mathbf{j}+3\,\mathbf{k}$

$2\mathbf{a}+3\mathbf{b}=2\,(2\,\mathbf{i}-4\,\mathbf{j}+4\,\mathbf{k})+3\,(2\,\mathbf{j}-\mathbf{k})=4\,\mathbf{i}-8\,\mathbf{j}+8\,\mathbf{k}+6\,\mathbf{j}-3\,\mathbf{k}=4\,\mathbf{i}-2\,\mathbf{j}+5\,\mathbf{k}$

$|\mathbf{a}|=\sqrt{2^2+(-4)^2+4^2}=\sqrt{36}=6$

$|\mathbf{a}-\mathbf{b}|=|(2\,\mathbf{i}-4\,\mathbf{j}+4\,\mathbf{k})-(2\,\mathbf{j}-\mathbf{k})|=|2\,\mathbf{i}-6\,\mathbf{j}+5\,\mathbf{k}|=\sqrt{2^2+(-6)^2+5^2}=\sqrt{65}$

26. Call the two tensile forces $\mathbf{T}_3$ and $\mathbf{T}_5$, corresponding to the ropes of length 3 m and 5 m. In terms of vertical and horizontal components,

$$\mathbf{T}_3=-|\mathbf{T}_3|\cos 52^\circ\mathbf{i}+|\mathbf{T}_3|\sin 52^\circ\mathbf{j}\quad (1)\qquad \text{and}\qquad \mathbf{T}_5=|\mathbf{T}_5|\cos 40^\circ\mathbf{i}+|\mathbf{T}_5|\sin 40^\circ\mathbf{j}\quad (2)$$

The resultant of these forces, $\mathbf{T}_3+\mathbf{T}_5$, counterbalances the force of gravity acting on the decoration [which is $-5g\,\mathbf{j}\approx -5(9.8)\,\mathbf{j}=-49\,\mathbf{j}$]. So $\mathbf{T}_3+\mathbf{T}_5=49\,\mathbf{j}$. Hence

$\mathbf{T}_3+\mathbf{T}_5=(-|\mathbf{T}_3|\cos 52^\circ+|\mathbf{T}_5|\cos 40^\circ)\,\mathbf{i}+(|\mathbf{T}_3|\sin 52^\circ+|\mathbf{T}_5|\sin 40^\circ)\,\mathbf{j}=49\,\mathbf{j}$. Thus

$-|\mathbf{T}_3|\cos 52^\circ+|\mathbf{T}_5|\cos 40^\circ=0$ and $|\mathbf{T}_3|\sin 52^\circ+|\mathbf{T}_5|\sin 40^\circ=49$.

From the first of these two equations $|\mathbf{T}_3|=|\mathbf{T}_5|\dfrac{\cos 40^\circ}{\cos 52^\circ}$. Substituting this into the second equation gives

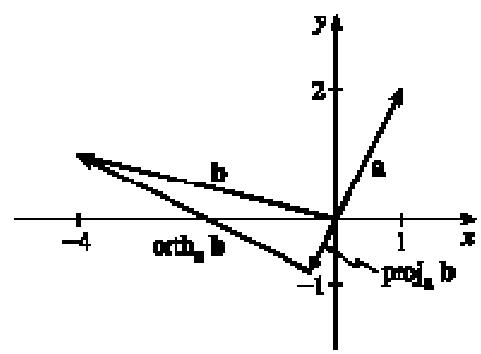
$|\mathbf{T}_5|=\dfrac{49}{\cos 40^\circ\tan 52^\circ+\sin 40^\circ}\approx 30$ N. Therefore, $|\mathbf{T}_3|=|\mathbf{T}_5|\dfrac{\cos 40^\circ}{\cos 52^\circ}\approx 38$ N. Finally, from (1) and (2),

$\mathbf{T}_3\approx -23\,\mathbf{i}+30\,\mathbf{j}$, and $\mathbf{T}_5\approx 23\,\mathbf{i}+19\,\mathbf{j}$.

9.3: 28, 34, 42

28. Using the formula in Exercise 27 and the result of Exercise 24, we have

$$\operatorname{orth}_{\mathbf{a}}\mathbf{b}=\mathbf{b}-\operatorname{proj}_{\mathbf{a}}\mathbf{b}=\langle -4,1\rangle-\left\langle -\tfrac{2}{5},-\tfrac{4}{5}\right\rangle=\left\langle -\tfrac{18}{5},\tfrac{9}{5}\right\rangle$$



34. Here $|\mathbf{D}|=100$ m, $|\mathbf{F}|=50$ N, and $\theta=30^\circ$. Thus $W=|\mathbf{F}|\,|\mathbf{D}|\cos\theta=(50)(100)\left(\frac{\sqrt{3}}{2}\right)=2500\sqrt{3}$ joules.

42. Let the figure be called quadrilateral $ABCD$. The diagonals can be represented by $\overrightarrow{AC}$ and $\overrightarrow{BD}$. $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$ and $\overrightarrow{BD} = \overrightarrow{BC} + \overrightarrow{CD} = \overrightarrow{BC} - \overrightarrow{DC} = \overrightarrow{BC} - \overrightarrow{AB}$ (Since opposite sides of the object are of the same length and parallel, $\overrightarrow{AB} = \overrightarrow{DC}$.) Thus

$$\begin{aligned}\overrightarrow{AC} \cdot \overrightarrow{BD} &= \left(\overrightarrow{AB} + \overrightarrow{BC}\right) \cdot \left(\overrightarrow{BC} - \overrightarrow{AB}\right) \\ &= \overrightarrow{AB} \cdot \left(\overrightarrow{BC} - \overrightarrow{AB}\right) + \overrightarrow{BC} \cdot \left(\overrightarrow{BC} - \overrightarrow{AB}\right) \\ &= \overrightarrow{AB} \cdot \overrightarrow{BC} - \left|\overrightarrow{AB}\right|^2 + \left|\overrightarrow{BC}\right|^2 - \overrightarrow{AB} \cdot \overrightarrow{BC} = \left|\overrightarrow{BC}\right|^2 - \left|\overrightarrow{AB}\right|^2\end{aligned}$$

But $\left|\overrightarrow{AB}\right|^2 = \left|\overrightarrow{BC}\right|^2$ because all sides of the quadrilateral are equal in length. Therefore $\overrightarrow{AC} \cdot \overrightarrow{BD} = 0$, and since both of these vectors are nonzero this tells us that the diagonals of the quadrilateral are perpendicular.